Mid exam Program Correctness, 8 march 2013

Problem 1 (15 pt). Given are the program variables a, $b : \mathbb{Z}$. Design an annotated program fragment S that consists of exactly two assignments that satisfies

$$\{a \cdot b = X \ \wedge \ a + 2 \cdot b = Y\}$$
$$S$$
$$\{a = X \ \wedge \ b = Y\}$$

Problem 2 (25 pt). Given is the program variable $n : \mathbb{Z}$. Design an annotated command S that satisfies

$$\{n = 4 \cdot X + 7 \ \vee \ n = 6 \cdot X + 4\}$$
$$S$$
$$\{n = X\}$$

Problem 3 (50 pt). The function $f : \mathbb{N} \to \mathbb{N}$ is defined by the recurrence:

$$n = 0 \ \Rightarrow \ f(n) = 0$$
$$n > 0 \ \Rightarrow \ f(n) = f(n - 1) + 2 \cdot n - 1$$

(a) (10 pt) Proof: $(\forall n \in \mathbb{N} : f(n) = n^2)$

(b) (40 pt) Design an annotated command T that satisfies

> **const** $x : \mathbb{N}$;
> **var** s, $z : \mathbb{N}$;
> $\quad \{P : \ x = X^2 \ \wedge \ X \geq 0\}$
> T
> $\quad \{Q : \ z = X\}$

Use a **while** loop with an invariant that contains at least the conjunct $s = f(z)$. You are only allowed to use addition, and multiplication by 2.